Quarter-Wavelength Coupled Dielectric Plate Resonators for High Selectivity TE_{10}-Mode Filters

ANDREW S. PODGORSKI, STUDENT MEMBER, IEEE, AND ROBERT H. MACPHIE, SENIOR MEMBER, IEEE

***Abstract*—An analysis of high selectivity TE_{10}-mode filters with quarter-wavelength coupled resonators formed by axially spaced dielectric plates is presented and shows that high-loaded quality factors of individual resonators can be obtained by placing the resonant frequency close to the waveguide cutoff frequency and by using low-loss low-dielectric constant materials. Design equations for Butterworth and Chebyschev filters are presented and employed in a three-cavity Butterworth filter having 30-MHz bandwidth at resonant frequency at 7250 MHz. Experimental results show that filter performance can be well predicted.**

THE OBJECTIVE of this paper is to call attention to a possible use of dielectric resonators in high-selectivity TE_{10}-mode Butterworth and Chebyschev filters. Filters of this type with moderately wide band widths have been reported recently [1], [2]. In such filters a basic resonator consists of two transverse dielectric walls of thickness $0<l_1 \leqslant (\lambda_{g1}/4)$ and with relative dielectric constant $\epsilon_{r1}=\epsilon'_{r1}-j\epsilon''_{r1}$, as shown in Fig. 1(a). The medium inside the resonator (between the walls) is of length $n(\lambda_{g2}/2) \leqslant l_2 < (3n/2)\lambda_{g2} (n=1,2,\cdots,N)$ and has a relative dielectric constant $\epsilon_{r2}=\epsilon'_{r2}-j\epsilon''_{r2}$, with $|\epsilon_{r2}| \ll |\epsilon_{r1}|$. The wave impedances of the walls and the interior are Z_1 and Z_2, respectively.

Using basic wave matrix theory for two-port networks to analyze a single dielectric resonator, the maximum value of external quality factor Q_e, which occurs for $l_1=\lambda_{g1}/4$ and $l_2=(\lambda_{g2}/2)n$, can be shown to be (see Fig. 1(b))

$$Q_{e,\max}=\frac{F_+ + F_-}{2|F_+ - F_-|} \tag{1}$$

where

$$F_\pm = \left[(\epsilon'_{r2}-K)\left(1 \pm \frac{T_0}{\pi n}\right)^2 + K\right]^{1/2}$$

$$T_0 = \tan^{-1}\left[2\left(\frac{Z_1}{Z_2}\right)^2\right]$$

$$\left(\frac{Z_1}{Z_2}\right)^2 = \frac{\epsilon'_{r2}-K}{\epsilon'_{r1}-K} \qquad K=\left(\frac{f_c}{f_0}\right)^2, \qquad n=1,2,\cdots,N$$

where f_c and f_0 represent the cutoff and resonance frequencies of the dielectric resonator, respectively.

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The authors are with the Department of Electrical Engineering, University of Waterloo, Waterloo, Ont., Canada N2L 3G1.

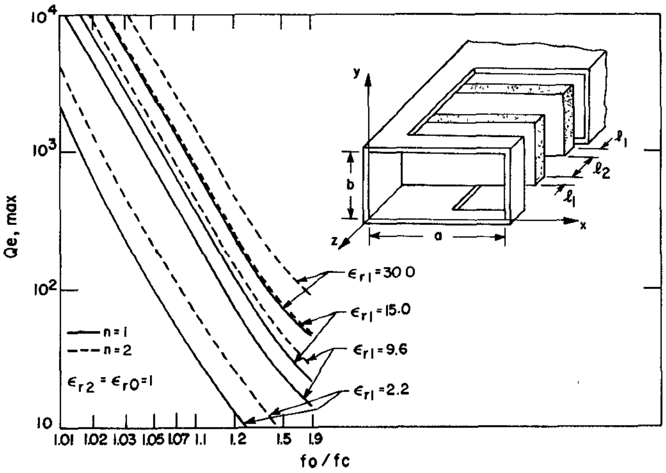

Fig. 1. (a) General view of the dielectric resonator. (b) The maximal external quality factor $Q_{e,\max}$ of dielectric cavity, as a function f_0/f_c, calculated for different mode factors n and different plate dielectric constants ϵ'_{r1}.

Equation (1) indicates that for $\epsilon'_{r2}=\epsilon_{r0}=1$, $Q_{e,\max}$ can be *increased indefinitely* by moving the cutoff frequency closer and closer to the resonance frequency. In order to obtain the loaded-quality factor Q_L, the external quality factor Q_e has to be modified by the addition of loss terms. This can be done by the calculation of small losses at resonance frequency using an equation derived by Cohn [3]. For a single resonator this equation simplifies to

$$Q_L = Q_e\left(1+\frac{L_{a0}}{4.343}\right)^{-1} \tag{2}$$

with L_{a0} representing the attenuation at resonance frequency, in decibels. For $Q_e=Q_{e,\max}$, L_{a0} can easily be found from the following equation:

$$L_{a0} = \cosh 2\phi_1 \cosh\phi_2 + \frac{1}{2}\left[\left(\frac{Z_1}{Z_2}+\frac{Z_2}{Z_1}\right)(\sinh\phi_2+\cosh\phi_2)\right]\sinh 2\phi_1 + \frac{1}{4}\left[\left(\frac{Z_1}{Z_2}-\frac{Z_2}{Z_1}\right)^2+\left(\frac{Z_1}{Z_2}+\frac{Z_2}{Z_1}\right)^2\cosh 2\phi_1\right]\sinh\phi_2 \tag{3}$$

where

$$\phi_1=\alpha_1 l_1,\ \phi_2=\alpha_2 l_2,\ \alpha_1=\alpha_{1C}+\alpha_{1D},\ \alpha_2=\alpha_{2C}+\alpha_{2D}.$$

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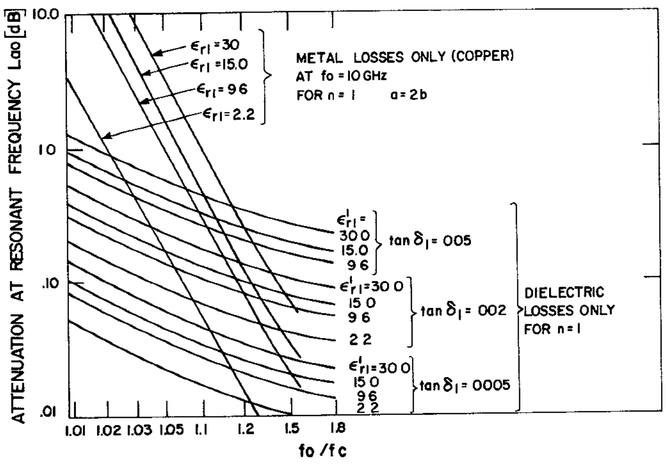


Fig. 2. Attenuation at resonant frequency, as a function of f_0/f_c, resulting from metal (copper) losses in the central part of cavity and dielectric losses of the dielectric plates.

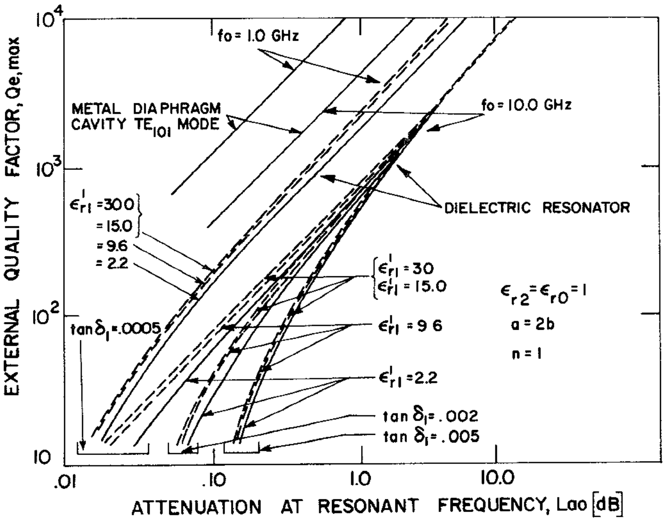


Fig. 3. Maximal external quality factor $Q_{e,\max}$ of a dielectric resonator as a function of attenuation at resonance Lao for different dielectric constant and loss tangents of dielectric plates, in comparison to metal diaphragm cavity operating at $f_0 = 1.5 f_c$ [5, p. 245].

In (3), attenuation constants α_{1C} and α_{2C} result from finite waveguide wall conductance in the presence of dielectric, whereas α_{1D} and α_{2D} are the attenuation constants resulting from the loss tangents of the dielectric materials.

To investigate the high-Q possibilities implicit in (1) we set the dielectric constant $\epsilon'_{r2} = \epsilon'_{r0} = 1$ (the cavity is air filled) and assume that the resonant frequency lies in the region specified by

$$\left\{ \frac{3\pi}{2} \frac{\epsilon'_{r2} \tan\delta_2}{\frac{f_c}{f_0}\left[\frac{a}{2b}\epsilon'_{r2} + \left(\frac{f_c}{f_0}\right)^2\right]} \right\}^2 \cdot \sigma \ll \underset{[\mathrm{GHz}]}{f_0} \ll \left\{ \frac{3\pi}{2} \frac{\epsilon'_{r1} \tan\delta_1}{\frac{f_c}{f_0}\left[\frac{a}{2b}\epsilon'_{r1} + \left(\frac{f_c}{f_0}\right)^2\right]} \right\}^2 \frac{\epsilon'_{r1}}{\epsilon'_{r2}} \sigma. \quad (4)$$

A small attenuation loss at resonance frequency (see Fig. 2) can be calculated by simple addition of attenuations resulting from dielectric losses ($\alpha_D = \alpha_{D1}$) and wall losses ($\alpha_C = \alpha_{C2}$). In (4) $\tan\delta_1$ and $\tan\delta_2$ are the loss tangents of dielectric walls and medium inside the resonator, respectively, and σ[mho/m] represents the conductance of the waveguide walls. Using (1) and (2) it is possible to show (Fig. 3) that a simple increase of dielectric constant ϵ'_{r1} does not necessarily result in an increase of external Q if loss in dielectric is taken into account. One can, therefore, conclude that since low-dielectric constant materials may have lower loss tangents it appears that a small attenuation at resonant frequency can be obtained by keeping the dielectric constant of the transverse dielectric walls low and shifting the cutoff frequency closer to the resonant frequency. Moreover since low-dielectric constant materials having low-loss tangent are easily available it should be possible to build high-selectivity dielectric resonators.

The cavity described above can easily be used to synthesize any N-cavity filter linked by quarter-wave sections of waveguide as in the Mumford concept of loaded Q for series and parallel resonators [4]. Maximally flat filters can be calculated according to the theories of Bennett and Darlington [4] where required loaded Q's of the resonators are

$$Q_{L_r} = Q_T \sin\left[\left(\frac{2r-1}{2N}\right)\pi\right] \quad (5)$$

and where $r = 1, 2, \cdots, N$ represents the resonator number, N is the number of resonators, and Q_T is the selectivity of the total filter. For realization of Chebyschev filters one can use a standard reference text [5] to calculate parameters g_r in the following formula for the loaded Q's of resonators

$$Q_{L_r} = \frac{1}{2} g_r \frac{\left[\frac{1}{2}(\lambda_{g01} + \lambda_{g02})\right]^3}{(\lambda_{g01} - \lambda_{g02})\lambda_0^2} \quad (6)$$

where

$$\lambda_0 = \frac{2c}{f_1 + f_2}$$

and $r = 1, 2, \cdots, N$.

In (6) f_1 and f_2 represent the lower and higher passband frequencies, respectively, λ_{g01} and λ_{g02} are their corresponding waveguide wavelengths, and c is the velocity of light.

In order to implement the loaded Q concept for filter design two basic functions which characterize a dielectric resonator are needed:

$$l_2 = F(l_1) \text{ at } f = f_0$$

gives the length l_2 between the dielectric walls of the resonator as a function of the wall thickness l_1;

$$Q_L = G(l_1)$$

is the loaded Q of the resonator as a function of the dielectric wall thickness l_1. For the lossy case both functions can be obtained by digital computer and graphs can be drawn [1]. For the lossless case a moderately accurate representation of $l_2 = F(l_1)$ can be obtained assuming that at resonance the input impedance is real:

$$l_2 = \frac{1}{\beta_2} \cdot \left\{ \tan^{-1}\left[\frac{2\left(\frac{Z_1}{Z_0} - \frac{Z_0}{Z_1}\right)\frac{1}{K_1}}{\left(\frac{Z_0}{Z_2} - \frac{Z_2}{Z_0}\right) + \left(\frac{Z_1^2}{Z_0 Z_2} - \frac{Z_0 Z_2}{Z_1^2}\right)\frac{1}{K_1^2}} \right] + n\pi \right\}. \quad (7)$$

For $Z_2/Z_1 \gg 1.5$ and for $0.23\lambda_{g1} > l_1 > 0.125\lambda_{g1}$ one can calculate the external Q from the following equation which uses the concept of lower (ω_-) and higher (ω_+) half-power frequencies:

$$Q = \frac{1}{2} \frac{\omega_+ + \omega_-}{|\omega_+ - \omega_-|} \quad (8)$$

with

$$\omega_{+,-} = \frac{c}{\epsilon'_{r2}} \cdot \sqrt{\left\{ \frac{1}{l_2}\left\{ \tan^{-1}\left[2\frac{Z_1}{Z_2}\left(K_1 \pm \frac{Z_1}{Z_2} S\right)\right] + \pi \cdot n \right\}\right\}^2 + \left\{\frac{\pi}{a}\right\}^2}$$

where

$$S = \sqrt{\frac{2}{T^2} - 1}$$

$$T = \sin^2(\beta_1 l_1)\cos\beta_2 l_2$$

$$K_1 = \cot\left\{ l_1 \sqrt{\left(\frac{\omega}{c}\right)^2 \epsilon'_{r1} - \left(\frac{\pi}{a}\right)^2} \right\}.$$

For increased accuracy the values of ω_- and ω_+ in (8) have to be calculated iteratively by first letting $\omega = \omega_0$, where ω_0 is the angular frequency at resonance; then one substitutes the calculated ω_+ and ω_- back as ω. Usually two iterations are enough to obtain reasonable accuracy. It should be pointed out that for constant f_0/f_c the small change of l_1 from $0.23\lambda_{g1}$ to $0.125\lambda_{g1}$ results in the value of Q_e increasing about threefold; a change of f_0/f_c from 1.8 to 1.05 increases the value of Q_e tenfold and an increase of the mode factor n by one almost doubles the value of Q_e.

In designing the quarter-wavelength coupled cavity filter one has to account for phase change resulting from insertion of the cavity. Only for $l_1 = \lambda_{g1}/4$ and for the lossless case is the phase change equal to $\pi(1+n)$; in any other case it is not and can be calculated for the lossless case as follows:

$$\phi = \tan^{-1}\left[\frac{2(K_1^2 - 1) - K_1 K_2\left(\frac{Z_1}{Z_2} + \frac{Z_2}{Z_1}\right)}{\frac{Z_1}{Z_2}\left[K_1\left(2 + \frac{Z_2}{Z_1}K_1K_2\right) - \frac{Z_1}{Z_2}K_2\right] + \frac{Z_2}{Z_1}\left[K_1\left(2 + \frac{Z_1}{Z_2}K_1K_2\right) - \frac{Z_2}{Z_1}K_2\right]} \right]^{-1} + \pi(1+n) \quad (9)$$

where

$$K_1 = \cot\beta_1 l_1$$

$$K_2 = \tan\beta_2 l_2$$

$$\beta_1 = \sqrt{\left(\frac{\omega}{c}\right)^2 \epsilon'_{r1} - \left(\frac{\pi}{a}\right)^2}$$

$$\beta_2 = \sqrt{\left(\frac{\omega}{c}\right)^2 \epsilon'_{r2} - \left(\frac{\pi}{a}\right)^2}$$

$$n = 1, 2, 3, \cdots.$$

The dielectric resonator has an infinite number of resonant and antiresonant frequencies. For $Q_{e,\max}$ the resonant frequencies occur approximately when the spacing between dielectric walls is a multiple of a half-wavelength, that is

$$f_n = \frac{nc}{2l_2}\left[\epsilon'_{r2} + \left(\frac{l_2}{na}\right)^2\right]^{1/2}$$

and the antiresonance frequency occurs when the spacing is an odd multiple of a quarter-wavelength, that is

$$f_n = \frac{(n+1)c}{4l_2}\left\{\epsilon'_{r2} + \left[\frac{2l_2}{(2n+1)a}\right]^2\right\}^{1/2}.$$

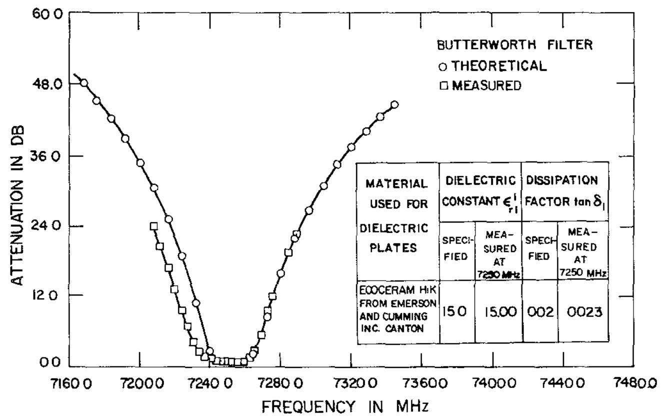
The attenuation (in decibels at antiresonance can be calculated for the lossless case and mode number $n = 1$ from the following approximate equation:

$$L_a \simeq 20\log_{10}\left[\frac{1}{2}\frac{\epsilon'_{r1} - K}{\epsilon'_{r2} - K}\right],$$

where

$$K = \left(\frac{f_c}{f_0}\right)^2. \quad (10)$$

In order to demonstrate the selectivity capabilities of dielectric resonators a three-cavity maximally flat (Butterworth) filter with passband between 7435 and 7265 GHz, and $f_0/f_c = 1.10, n = 1$, was designed (using a computer program developed previously [1]), built and tested. The data describing the dielectric material used to build the resonators is specified in Fig. 4. The theoretical



MATERIAL USED FOR DIELECTRIC PLATES	DIELECTRIC CONSTANT ϵ'_{r1}		DISSIPATION FACTOR $\tan \delta_1$	
	SPECIFIED	MEASURED AT 7250 MHz	SPECIFIED	MEASURED AT 7250 MHz
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Fig. 4. Theoretical and measured attenuation curves for a three-cavity Butterworth filter and specification of dielectric plate materials.

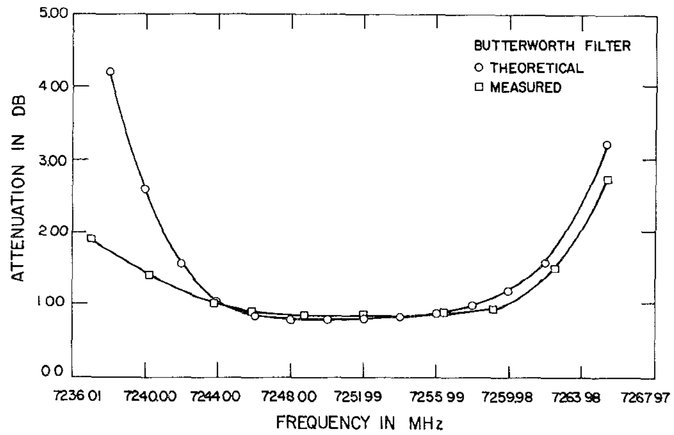

Fig. 5. Theoretical and measured attenuation curves in the passband for the filter from Fig. 4.

and measured attenuation curves are given in Figs. 4 and 5. Comparing the theoretical (lossy case) and measured frequency response from Figs. 4 and 5 we can see that the actual filter performs well, giving the minimum attenuation at resonance of 0.9 dB. The manufacturing tolerances for dimensions of the dielectric plates inserted into waveguide are extremely rigorous (± 0.001 mm) and measured by special metrology equipment with accuracy ± 0.00025 mm. Comparing the theoretical and measured values of the central frequency and loaded Q's of individual resonators maximum errors of ± 0.01 percent and 10 percent, respectively, were measured. The excess in the measured errors are attributed to the tolerances of the waveguide dimensions and the positioning and thickness errors of the dielectric plates.

In closing, the authors suggest that the high-selectivity dielectric filters introduced in this paper will provide a useful alternative to conventional diaphragm filters and that the use of low-loss temperature resistant dielectric materials for the resonator walls will increase their power handling capacity significantly.

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References

[1] A. S. Podgorski, "TE_{10} mode filters with quarter wavelength coupled resonators formed by axially spaced dielectric plates," M.A.Sc. thesis, University of Waterloo, Waterloo, Ont., Canada, Aug. 1975.
[2] S. Kuwano and K. Kokubun, "A class of maximally-flat waveguide bandpass filter consisting of array of dielectric slabs," *Trans. Inst. Electron. Commun. Eng. Jap. Sect. E* (Japan), vol. J61-B N, pp. 543–544, June 1978.
[3] S. B. Cohn, "Dissipation loss in multiple-coupled resonator filters," *Proc. IRE*, vol. 47, pp. 1342–1348, Aug. 1959.
[4] W. M. Mumford, "Maximally flat filters in waveguides," *Bell Syst. Tech. J.*, vol. 27, pp. 684–714, Oct. 1948.
[5] G. L. Mattaei, Leo Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks and Coupling Structures*. New York: McGraw-Hill, 1964, ch. 4, p. 99.